**Homework 10 – F2018**

**P9.42** Determine the energy stored in the circuit of Figure P9.42 in the dc steady-state.

**Solution:** *Leq* = 2 + 4 – 2 = 4 H. The circuit under steady-state conditions is a shown in Figure P9.42A, with inductors acting as short circuits and capacitors as open circuits. The two 1 Ω resistors combine in parallel into a 0.5 Ω resistor, and the two 1 F capacitors combine in parallel into a 2 F capacitor. The voltage across this capacitor is 0.5 V, so that the energy stored is 0.5×2×(0.5)2 = 0.25 J. The energy stored in the 2 H inductor is 0.5×2×(1)2 = 1 J. The energy stored in the 4 H inductor is 0.5×4×(0.5)2 = 0.5 J. The total energy stored is: 0.25 +1 + 0.5 = 1.75 J.

**P9.49** Derive TEC looking into terminal ‘ab’ in Figure P9.49.

**Solution:** Replacing the linear transformer by its T-equivalent circuit (Figure P9.49A), it is seen that on open circuit, the -*j*40 Ω in series with *j*10 Ω gives -*j*30 Ω. In parallel with *j*30 Ω, this results in an open circuit. The current through the *j*5 Ω is zero, so that both terminals of this inductor are at a voltage 5**IX**. The current in the -*j*5 Ω impedance is **ISRC** – **Ix**. From KVL, 10**IX** + *j*5(**ISRC** – **IX**) = 5**IX**. This gives  A, and 5**IX** = 20 V. It follows that **VTh** = **Vab** =  = 26.67 V.

On short circuit (Figure P9.49B), the voltage of the dependent source is still 20 V. (*j*10||*j*30) = *j*7.5 Ω. The current through the *j*5 Ω impedance is 20/(*j*12.5) A. From current division, **ISC** is this current multiplied by 3/4. This gives **ISC** = -*j*1.2 A. It follows that *ZTh* = (80/3)/(-*j*1.2) = *j*200/9 = *j*22.22 Ω.

**P9.51** Determine *kX* in Figure P9.51 so that no power is delivered or absorbed by *vSRC*2, given that *vSRC*1(*t*) = 10cos10*t* V, *vSRC*2(*t*) = 10sin10*t* V and *k* = 0.5.

**Solution:** If no power is delivered or absorbed by *vSRC*2, the current through the source must be zero. The circuit in the frequency domain is as shown in Figure P9.51A, where. *ωL* = 10×1 = 10 Ω;

-1/*ωC* = -1/10×0.01 = -10 Ω; 10sin10*t* = 10cos(10*t* – 90°), *jωM* = Ω.  A. It follows that -*j*10 = 1(*j*10 – *j*10) – *j*5 + *j*5 – , which gives *kX* = 1.

**P9.57** Derive TEC looking into terminals ‘ab’ in Figure P9.57, given that V. Represent Thevenin’s voltage in the time domain and express *ZTh* in rectangular coordinates.

**Solution:** Using the T-equivalent circuit becomes as shown in Figure P9.57A, with (*L*1 – *M*) = 1 H and (*L*2 – *M*) = 0. From KVL, the voltage across the lower *j*Ω inductor is (1 – *j*)**IO** and the downward current through this inductor is (1 – *j*)**IO**/*j* = (-1 – *j*)**IO**. The current through the upper *j*Ω inductor is (-2 – *j*)**IO** in the direction shown. From KVL about the outer loop, not including the current source, **VSRC** = *j*(-2 – *j*)**IO** + **IO** = 2(1 – *j*)**IO**. It follows that **IO** = **VTh** = , which in the time domain is *vTH*(*t*) = 2cos(*t* + 90°) = -2sin*t* V. On short circuit, **IO** = 0, so that both dependent sources are set to zero, and the *j* Ω inductor in the middle is short circuited. It follows that **ISC** = **VSRC**/*j*. Hence, *ZTh* =   Ω.

**P9.60** Determine **VO** and **VS** in Figure P9.60.

**Solution:** The mesh current equation for mesh 1 in Figure P9.60A is:

(4 + *j*3)**I1** – 2**I2** – *j*3**I3** – *j***I3** = 0, where **I3** = 2∠45° A, and **I2** = 0.5**VX** = 0.5×2(**I1** – **I2**) = **I1** – 0.5**VX**, or, **I1** = **VX** = 2**I2**; substituting in the mesh-current equation, (3 + *j*3)**I1** = *j*8∠45°, or,  and  V.

KVL around mesh 2 gives: **VX** – *j*2(**I3** – **I2**)

– *j*4**I2** + *j*2**I3** – **VS** = 0, or **VS** = **VX** – *j*2**I2** = 2(1 – *j*)**I2** = (1 – *j*)**I1** =  V.

**P9.66** Derive Thevenin’s equivalent circuit between terminals ‘ab’ in Figure P9.66, assuming that *vSRC*(*t*) = 5cos*ωt* V,

**Solution:** The circuit in the frequency domain is as shown in Figure 9.66A. *M* = . It is evident from equality of impedance in the two branches that **I1** = **I2** = **I**. This can be readily confirmed by taking KVL around the mesh on the RHS: ,

or, , which gives **I1** = **I2** = **I**. Hence, the voltage induced in either coil is *jωL***I** – *jωL***I** = 0, so that each coil can be replaced by a short circuit. The circuit becomes a shown in Figure P9.66Ba. It follows that that **VTh** = **Vac** = **VSRC** = 5cos*ωt* V.

When a test source is applied between terminals ‘ab’, with the independent source set to zero, the same argument can be applied to show that each inductive branch is equivalent to a short circuit, so that that *ZTh* = 0. Alternatively, the circuit can be redrawn as shown in Figure P9.66Bb. When the coupled coils are replaced by the T-equivalent circuit, *L*1 – *M* = *L*2 – *M* = *L* – *M* = 0, so that the source sees a short circuit.



**P10.12** Derive TEC looking into terminals ‘ab’ in Figure P10.12.

**Solution:** On open circuit, no current flows in the circuit (Figure P10.12A), The 10∠0° V appears across the primary winding, so that the voltage across the secondary winding is 40∠0° V. this is the same as **Vab** and hence and hence **VTh**.

 If a test voltage is applied, with the source set to zero, and the current through the test source is **IT** (Figure P10.12A), the current through the transformer secondary Is 4**IT**. The current through the lower 2 Ω resistor is 3**IT**, and the primary voltage is 14**IT**. The primary voltage is 56**IT**, so that **VT** = 50**IT**. It follows that *ZTh* = 50 Ω.

**P10.22** Derive TEC looking into terminal ‘ab’ in Figure P10.22.

**Solution:** On open circuit (Figure P10.22A), if the secondary current is **I2**, the primary current is **I2**/2. The primary voltage is 2**VTh**, and the source current is **I2**/2. From KVL, the voltages across the capacitors cancel out, so that 2**VTh**, = 12, and **VTh** = 6 V.

 On short circuit, the primary voltage is zero, the 2∠0° V appears across the capacitor on the RHS, resulting in a current of *j*0.2 A through this capacitor. The voltage across the capacitor on the LHS is 10∠0° V, resulting in a current 10/(-*j*10) = *j* A through this capacitor. The current through the 2∠0° V source is *j*0.8 A. The secondary current is *j*0.4 A, so that **ISC** *= j*1.2 A.It follows that *ZTh* = 6/*j*1.2 = -*j*5 Ω.

**P10.28** Derive TEC looking into terminals ‘ab’ in Figure P10.28.

**Solution:** On open circuit (Figure P10.28A), the transformer secondary current is **IO**, the primary current is 0.5**IO**, and the source current is 1.5**IO**. From KVL around the mesh on the LHS, 12 = 3**IO** + 2**V2** + **IO**, or, **V2** + 2**IO** = 6; from KVL around the mesh on the RHS, **IO** + **V2** = 5**IO**, or **V2** = 4**IO**. Substituting for **V2** gives **IO** = 1 A and **VTh** = 2∠0° V.

 On short circuit, **IO** = 0, which sets both independent sources to zero. The circuit reduces to that shown in Figure P10.28B. The source and 2 Ω resistance, referred to the secondary are 6 V in series with 2/4 = 0.5 Ω. This gives: **ISC** = 6/3.5 = 12/7 A. It follows that *ZTh* = 2/(12/7) = 7/6 Ω.